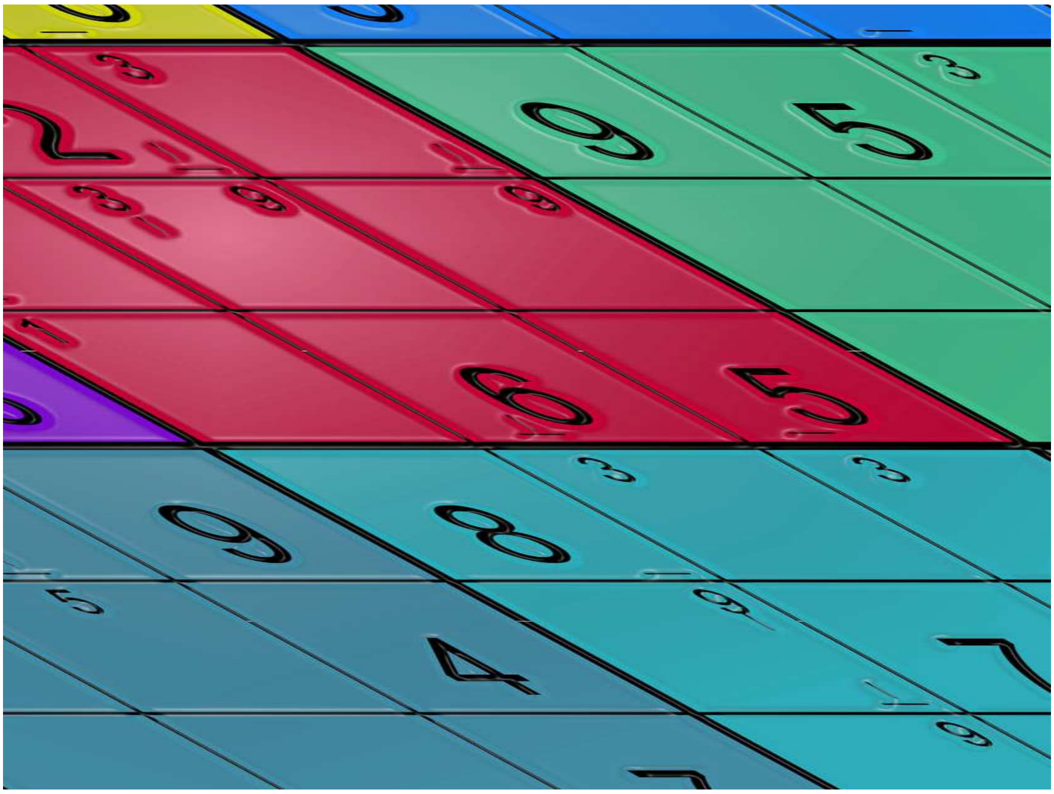
Links Method

Solve hard sudokus in less time

[Extract]

Gus Coyote



Contents

1.- INTRODUCTION

The origin of sudoku is uncertain. Everything seems to indicate that the first sudoku, as we know at present, was devised in the 60s by an architect from Indianapolis whose name was Howard Garns.

However, its roots started before that. In fact, sudokus are a variation of Latin squares, which are defined as square boards divided in cells, having each row and each column the same set of symbols or numbers (Fig. 1). In 17th century the famous Swiss mathematician Leonhard Euler described Latin squares as a mathematical curiosity related to his works in probability calculus.

3	1	4	2	5
4	2	5	3	1
5	3	1	4	2
1	4	2	5	3
2	5	3	1	4

Figure 1. Example of Latin square

In the 60s Howard Garns spent his free times in the Technical Office of Dagget, an architecture company, to devise new kinds of puzzles, his genuine and lonely passion. Starting from Latin square concept he added a new rule, to divide a 9x9 board in 9 square regions of 3x3 cells each one, so that as well as containing the same numbers (1 to 9) rows and columns, also regions would satisfy this requirement. Including certain quantity of initial numbers and starting from those simple rules, the aim would be to complete empty cells using deductive reasoning. Garns called the new game “Number Place” (that has also been traslated as “every number in its right place”).

After retiring, in 1979 he decided to publish the new game in a specialized magazine. The May edition in that year of Dell Pencil Puzzles and Word Games included the first brand sudoku known, although the name of Number Place was kept. It is assumed that It was completely unnoticed.

Of course, the theory that declares Japan as the birthplace of sudoku is dismantled. In fact, the only thing of current sudoku with Japanese origin is the name. However they were the first in realize the huge potential it had. In 1984, the leading puzzle design company in Japan, Nikoli, finds Number Place in Dell magazine and decides to import it for the puzzle fans in their country. Initially, the game was named “Suuji Wa Dokushin Ni Kagiru”, “numbers must exist just once” and soon became very popular. Nevertheless the name was too long, so that Kaji Maki, Nikoli chairman, abbreviated it as “sudoku”. It comes from “Su”, number or digit, and “Doku”, single, lonely.

In 1986 Nikoli introduced two innovations in the puzzle: the quantity of given numbers would be restricted to 30 at the most and puzzles would be symmetric (cells with given numbers would be distributed with central symmetry, Fig. 2), despite the second one is not always satisfied in present sudokus.

	9			5	8		3	
2		4	6	1				
	3		9					
								7
5	2	8				9	4	6
6								
					9		1	
				8	5	4		2
	5		3	4			6	

Figure 2. Symmetric sudoku (central symmetry)

In 1989, Loadstar/Softdisk Publishing published DigitHunt on the Commodore 64, which was apparently the first home computer version of Sudoku.

A long time later, in 1997, a New Zealander judge called Wayne Gould, during his holiday time in Japan found a sudoku magazine. Gould´s hobby was computer programming, and decided to write a sudoku random-generator program of different difficulty levels. He got some puzzles ready and visited London trying to convince the editors of *The Times* to publish them, demanding no money.

Seeing its increasing popularity in Japan, *The Times* finally decides to provide an opportunity to the new puzzle. In November 12th, 2004 is published the first sudoku in this journal. Subsequently the publishing of Sudoku in the *London Times* was just the beginning of an enormous phenomenon which swiftly spread all over Britain and its affiliate countries of Australia and New Zealand. Three days later *The Daily Mail* began publishing Sudoku puzzles titled as "Codenumber". By the end of May 2005 the puzzle was regularly published in many national newspapers in the UK, including *The Daily Telegraph, The Independent, The Guardian, The Sun* and *The Daily Mirror*. From then until now we know how popular become this fantastic game.

Why sudoku has reached such a high level of acceptance? Because it is a challenge. Because it makes us feel satisfied when we achieve the objective. Because it exercises the ability to concentrate. Because it is not necessary to do mathematical calculations. Because in every sudoku we face the fascinating uncertainty of whether or not we will be able to reach the final.

This manual is intended for those who already know the exciting world of sudoku, who have tried and liked it, but want more, and certainly hope to resolve sudokus in a reasonable time, specially those classified as "difficult" or "very difficult" . It is based on the resolution step by step of practical examples that can go along gradually, so that has been deliberately omitted to deepen the theoretical aspects, which I believe would not contribute much to the target.

We will focus on conventional sudoku, the 9x9, although there are other variants of the game as we know it. We are going to use a sudoku on paper, pencil and eraser, because as well as placing the numbers on the empty cells, it will be necessary to make other marks and deleting them when appropriate.

Although they are known to all, it is worth to review briefly the 5 sudoku rules: **1.-** You have to fill the empty cells with a single number from 1 to 9. **2.-** In the same row you can not have repeated numbers. **3.-** In the same column you can not have repeated numbers. **4.-** In the same region you can not have repeated numbers. **5.-** The solution of a sudoku is unique.

One of the questions I have had in mind since I know this game is what determines the degree of difficulty. At first I imagined that the quantity of given numbers would be a fundamental fact, but later I realized my mistake. There are relatively easy sudokus with few given numbers and challenging sudokus with many of them. As for the symmetrical and non symmetrical, experience tells me that within the more difficult ones it is more common to find the first. Most solved examples included in this manual (all except the second) have central symmetry. Definitively, experts say really to classify the level of difficulty of a sudoku, the techniques necessary for resolution must be taken into account.

2.- DEFINITIONS

Sudoku elements are shown at Fig. 3.

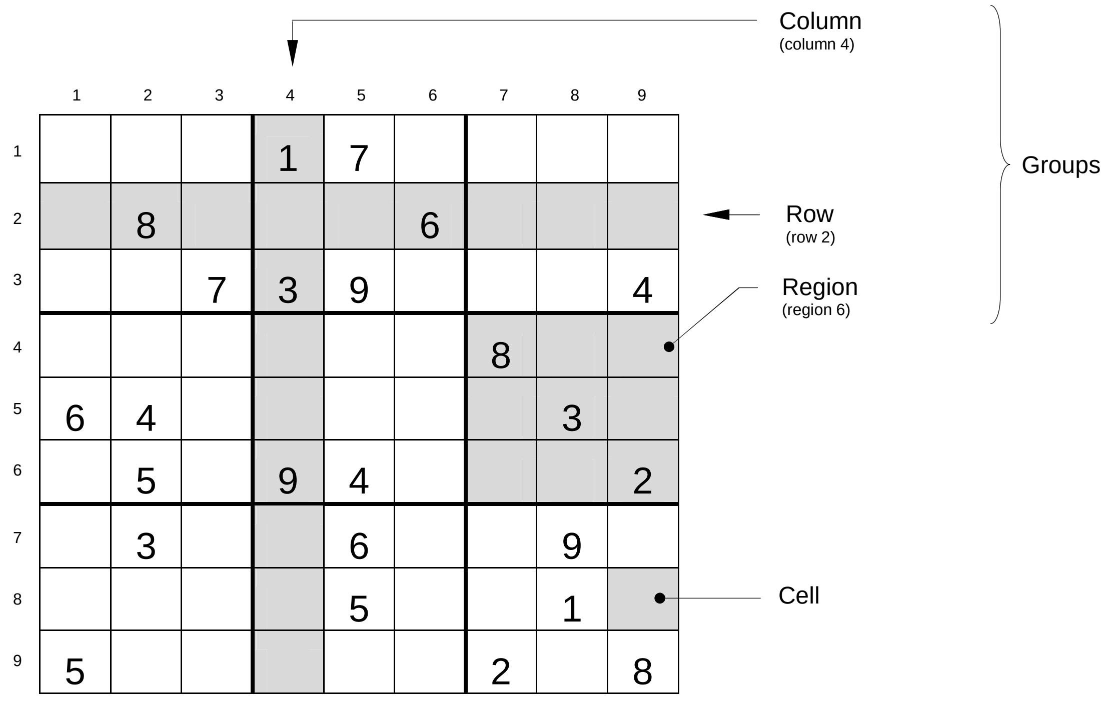


Figure 3. Sudoku elements

- **Board**: grid of 9x9 cells, that is to say 81 cells.

- **Cell**: individual grid element that can contain the numbers 1 to 9. Each cell is inserted in a row, in a column and a region simultaneously.

- **Row**: Horizontal 9 cell line.

- **Column**: Vertical 9 cell line.

- **Region or box**: 3x3 grid of cells (9 cells). We will number the regions 1 to 9 as shown in Fig. 4.

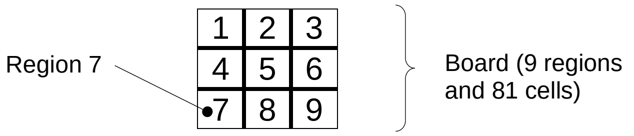


Figure 4. Region numbering

- **Group**: Row, column or region. There are 27 groups in a sudoku.

- **Place**: Write a final number, with complete certainty, in an empty cell.

3.- WHAT TO DO AND WHAT NOT TO DO

These are the guidelines of what to do to solve sudokus:

1.- Use common sense. I refer simply to take into account all the time the simple logic that everybody knows. As easy as that. But don’t lower your guard!

2.- Use all means at your disposal. Welcome any tool, manoeuvre, ruse or subterfuge that can provide some information, however small it may be, in the intricate task of solving a sudoku. Let's say bringing out "latent information" will give a result sooner or later.

3.- Be organized and methodical. It is important to have an strategy and implement it step by step. Of course, you must also assume some flexibility, but at the same time a general order should be maintained in order to progress more easily during the process.

And now, what not to do:

1.- Trial and error. Some believe that solving a Sudoku is a matter of trial and error. You start to put all the numbers that can "logically" until you can not continue like that and start with trial and error procedure. <<I’ll try placing number four here, and I’ll check>>. Wrong way. You have not to do this, except in very exceptional circumstances.

2.- Make a mistake. This issue is important because everyone will eventually slip up, and in the world of sudoku one mistake is almost always fatal. Don’t worry, when you realize that you were wrong it is prudent to quit and start again from the beginning, unless the error were very recent and can be amended to continue. In any case, the basic idea is worth investing a little time to check each step we are taking to avoid disappointing consequences afterwards.

4.- LINKS METHOD. INTRODUCTION

When facing a sudoku, approximately 80% of the cells are empty and we must be able to fill them all as best as we can. This does not seem so easy, and indeed there is nothing simple (that is precisely the greatness of this game).

With the links method we have to use the usual techniques. The difference is to put more emphasis on applying the following proverb:

"A short pencil is better than a long memory."

In other words, we have paper, pencil and eraser. Let's use them! Let's write and delete more, let's use as much as possible the chances of pencilmarking right for to be aware of them at any time with a glance. And let's erase all that is no longer useful to avoid more complication than we already have.

Someone will say: <<Yes, but writing and erasing means wasting time>>. True, but you lose a lot more time trying to remember or reviewing things that are not sure.

In fact, many fans of sudoku mark provisional little numbers in empty cells to show potential candidates. Other, write dots that are translated into numbers according to their position. This makes it much easier because it allows to bring out information that otherwise would remain there but it would not be so obvious.

Well, let's go a little further. Mark with a pencil those little numbers, but also the relations between them, links to associate with each other and will show clues that soon or later are going to help, and how!.

Procedure of sudoku solving has different levels of possibilities grouping. The basic level is the unit, by which we can locate individual final numbers using logic. If we fail to complete the sudoku no choice but to go to multiple levels, in which we must consider pairs, trios, quads, etc.

However, in links method the basic or initial level consists of units and also in pairs, which increases its power significantly. Therefore will be much less frequent the need to level up to use trios, quads, etc. Links are not anything but how to establish those pairs.

This is just adding small pencil strokes to candidate numbers marked in empty cells, so that we can quickly identify the numbers that are linked, the cells on which the link exists, the groups (rows, columns or regions) which is the link and, as we will see, the direction stablished by link.

Links method is applicable to any level of difficulty, but you are advised to use it especially in medium-high and high complexity sudokus, because in easy ones can be faster dispense with pencilmarks and solve “in head”.

5.- BASIC TECHNIQUES

A more detailed explanation of the basic techniques are described, on the fly, during resolution of sudoku 1.

- **Naked single cell**: if in a cell has been ruled out all numbers except one, then it must be placed in that cell.

- **Hidden single cell**: if in a particular group (row, column or region), a number have been excluded from all cells except one, then it must be placed in that cell.

- **Pencilmarks**: provisional small numbers and strokes written on top of the empty cells to help stablish possibilities and links that facilitate the resolution of sudoku. Pencilmarks can be written, edited or deleted, and there are three types: links, candidates and naked pairs.

- **Links**: also called **marked conjugate pairs**, are marks which indicate that a number can only be in two empty cells within a group. Are shown with this number and a small stroke in each linked cell, pointing each stroke to the other cell.

3		3

It is also common to find closed linkages of four elements with the links of the same number and are called x-wing, as we will see.

- **Candidates**: marks which consist of all possible numbers that may be placed into an empty cell. Always are written underlined.

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- **Naked pairs**: are candidates and links at the same time. When two empty cells in the same group have 2 candidates each one and are the same in both, these candidates are also interlinked and make up a naked pair. Numbers are indicated by underlining and twin strokes pointing to the other cell. Naked pairs are not deleted or modified, are just waiting to know which of the only two options is correct to place the definitive numbers in both cells. The groups to which they belong can not contain other cells with either of the two candidates of naked pair.

48 =	= 48

- **Counting**: You can count the numbers of a group (row, column or region) and check how many cells are empty. If no, the group is complete. If there is one, must be immediately completed the missing number. If there are two, a naked pair must be pencilmarked. If 3 or more, you have the ability to pencilmark links, candidates or naked pairs. Later we will see that also is possible to make crossed counting on individual cells (crossing rows, columns and regions).

- **Scanning**: Rule out empty cells for a specific number within a group by the positions of that number in other groups. If after scanning an only empty cell is not ruled out, is located in it the definitive number, and if there are two, we pencilmark a link. In addition to using numbers for other groups to scan, we can also use links and naked pairs if they have the right direction.

The most intuitive is scanning on regions, but also can be scanned rows and columns. If after region scanning 3 cells are empty without ruling out, directly we can not locate or pencilmark, but sometimes we can get some clues: if empty cells not ruled out are aligned vertically or horizontally, are also setting a scan direction. If not aligned, it is possible to scan and pencilmark a link in rows or columns that cross that region, particularly in wich share whith it a single empty cell not ruled out.

6.- METODOLOGY WITH LINKS

In the standard methodology, sudokus can have 3 different stages in the process of resolution. They are:

- Stage 1: **Basic techniques**, ie, scanning and counting in order to place as numbers as possible.

- Stage 2: **Candidates pencilmarking and use of advanced techniques**, in addition to the basic techniques. Only used if stage 1 is not sufficient to solve the sudoku.

- Stage 3: **"Brute force" method** or trial and error. This method is used only rarely, if stage 2 is not sufficient for full resolution.

To solve a sudoku using methodology with links we set the criterion that the more information we get, the more easily we can advance. Thus in the initial stages we will use the tools more useful and fast. If it is not enough we must to use other methods, but with the advantage of having enough information already "in sight".

Methodology with links is the same as the standard, except that in stage 1 we will use the basic techniques, scanning and counting in order to place as numbers as possible **and also in order to pencilmark all the links and naked pairs as possible.**

In second and third stages, if necessary, the only difference is that we will have marked a lot of information to make process easier, but otherwise there is no distinction regarding the standard methodology.

The advantage of the links is that many of the hard sudokus will be able to resolve in the first stage, without having to pencilmark the candidates and without using advanced techniques. This will allow to save time and finish before than using the standard methodology.

Therefore, the techniques used in the first stage typical of the links method are as follows:

- Scanning, one or more times, on each of the 9 numbers.
- Counting of groups containing up to 4 empty cells.
- Placing of all definitive numbers as possible.
- Pencilmarking of all links as possible.
- Pencilmarking of all naked pairs as possible.
- As a last resort, crossed counting of groups containing up to 5 empty cells.

Moreover, regardless of the naked pairs, we can pencilmark the simple candidates of 2 elements found.

Using these basic tools, to solve a sudoku is simply a sequence of actions leading to consequences, which lead to more consequences, and so on, until the puzzle is solved entirely.

What has been seen so far has little value unless illustrated with examples. So without further delay, let the first one.

7.- SUDOKU #1

Let's start with the puzzle of Fig. 5.

	9			5	8		3	
2		4	6	1				
	3		9					
								7
5	2	8				9	4	6
6								
					9		1	
				8	5	4		2
	5		3	4			6	

Figure 5.

As previously mentioned, links method is based on squeezing the most out of the first stage, which will try to place all possible definitive numbers, and also obtain the maximum amount of links and naked pairs, all using scanning and counting .

The quickest way is to start scanning on regions each one of the nine numbers. We begin with a number and focus on all regions in which there it doesn't exist to check on each of them how many cells remain unscanned. If only one remains, we will place a definitive. If we get two, we pencilmark a link. In case of getting 3, we may get a clue. If we get 4 or more, we can not do anything. Once exhausted the options of that number we continue with the next.

We could for example start at 1, then 2 and so on until 9. However, we must to start with the easier numbers, because, to put it in some way, the sooner we get information, the easier to get new information later. A priori we do not know which of the given numbers are more or less easy to try to complete, but we do know that the most common are, the more likely to be easier.

So the first step is to determine the frequency of given numbers:

1 appears 2 times; 2, 3 times; 3, 3 times; 4, 4 times; 5, 4 times; 6, 4 times; 7, 1 time; 8, 3 times; 9, 4 times. Therefore, a proper order to start the scans will be: 4, 5, 6, 9, 2, 3, 8, 1, 7.

Scanning of number 4. Links and X-wing

Let's start scanning the number 4. We will focus on regions that do not have that number, which in this case are 2, 3, 4, 5 and 7. In each of these regions we will mentally rule out all those cells that are affected vertically or horizontally by another 4 and then will check how many empty cells are not ruled out.

For example, consider in Fig. 5 the region 5, which is central and is completely empty. Vertically, we see that the number 4 in the region 8 scan up 3 cells in region 5, which are ruled out. Also, the number 4 in region 6 scan to the left 3 cells (although one was already ruled out) in region 5. How many cells without scanning, ie not ruled out, are remaining in the region 5? Are remaining 4 cells (the 4 corners) and therefore we can not do anything.

The best way to do it is to think in imaginary horizontal and vertical lines that start in all the numbers 4 that there are in the puzzle and end crossing out (sweeping or scanning) cells in the corresponding region.

Let’s focus now in region 7 of Fig. 5. There are two numbers 4 in regions 8 and 9 that scan to the left a total of 6 cells in the region 7. 3 cells are currently unscanned. In addition, in region 1 there is another 4 that scans down over one of the remaining 3 cells. Well done! We have only 2 unscanned cells in Region 7. We can write our first **link** (Fig. 6).

	9			5	8		3	
2		4	6	1				
	3		9					
								7
5	2	8				9	4	6
6								
4	4				9		1	
				8	5	4		2
	5		3	4			6	

Figure 6.

In the two cells left without being scanned by any number 4 we have pencilmarked a link, horizontal in this case (Fig. 6). Also there are vertical and oblique. This link is telling us the following:

- In region 7, the number 4 can only be in one of the two cells of the link. In the remaining blank cells of that region is impossible to have a 4.

- In row 7, the number 4 can only be in one of the two cells of the link. In the remaining blank cells of that row is impossible to have a 4.

- When placing a number different of 4 in either of the two linked cells, we can automatically locate the number 4 in the other.

- Linked cells stablish a scanning direction of number 4 through row 7, to right side. In some cases it might be very useful.

As we see, the auxiliary numbers (pencilmarks) that indicate a link are written to a smaller size in the top of the cells. The strokes that mark the horizontal links are written above those numbers.

Now we also scan digit 4 on the region 3. In Fig. 6 we note that region 3 has two numbers 4 below (regions 6 and 9) and one on the left (region 1). As before, when ruling out the empty cells that are scanned by these numbers, there are just 2 cells left in region 3. We can pencilmark a new link of number 4, vertical in this case (Fig. 7).

	9			5	8		3	4
2		4	6	1				
	3		9					4
								7
5	2	8				9	4	6
6								
4	4				9		1	
				8	5	4		2
	5		3	4			6	

Figure 7

Likewise, new link have also consequences:

- In region 3, the number 4 can only be in one of the two cells of the link. In the remaining blank cells of that region is impossible to have a 4.

- In column 9, the number 4 can only be in one of the two cells of the link. In the remaining blank cells of that column is impossible to have a 4.

- When placing a number different of 4 in either of the two linked cells, we can automatically locate the number 4 in the other.

- Linked cells stablish a scanning direction of number 4 through column 9, downwards. In some cases it might be very useful.

Let’s see now region 2. Here we can scan with 4 from region 1 and with 4 from region 8. Also there are left two empty cells unscanned, therefore we pencilmark the corresponding link, which in this case is not vertical or horizontal, but oblique (Fig. 8).

	9		(4)	5	8		3	(4)
2		4	6	1				
	3		9		(4)			(4)
								7
5	2	8				9	4	6
6								
(4)	(4)				9		1	
				8	5	4		2
	5		3	4			6	

Figure 8.

The consequences will be similar to the above but with some variation, since being oblique link does not affect any row or column, neither stablish scanning direction. These are:

- In region 2, the number 4 can only be in one of the two cells of the link. In the remaining blank cells of that region is impossible to have a 4.

- When placing a number different of 4 in either of the two linked cells, we can automatically locate the number 4 in the other.

Before we continue let's look at an interesting detail. Look at Fig. 8 the regions 2 and 3 with the links just marked. As it happens, both links are on rows 1 and 3. This does not bring us so much, but think about the following: In region 2, number 4 will be in row 1 or row 3. On the other hand, in region 3, the number 4 will be also in row 1 or row 3. So far without incident. But, what if in region 2 the number 4 is finally in first row? Where will be placed the number 4 of region 3? Obviously, in the third row. Conversely, if in region 2 the number 4 is finally placed on row 3, in region 3 should be located on row 1. To see it more clearly, are shown in Fig. 9 the only two possibilities that may occur.

If we observe it from the point of view of row 1 (see row 1 in the two illustrations of Fig. 9), the number 4 can only be placed in only two cells. And the same with row 3.

We've just found 2 new links that "close the loop" of the links found earlier. Put another way, we've just found the first **x-wing**. Resulting table is shown in Fig. 10. To better identify the closed circuit has been provisionally represented by dashed lines.

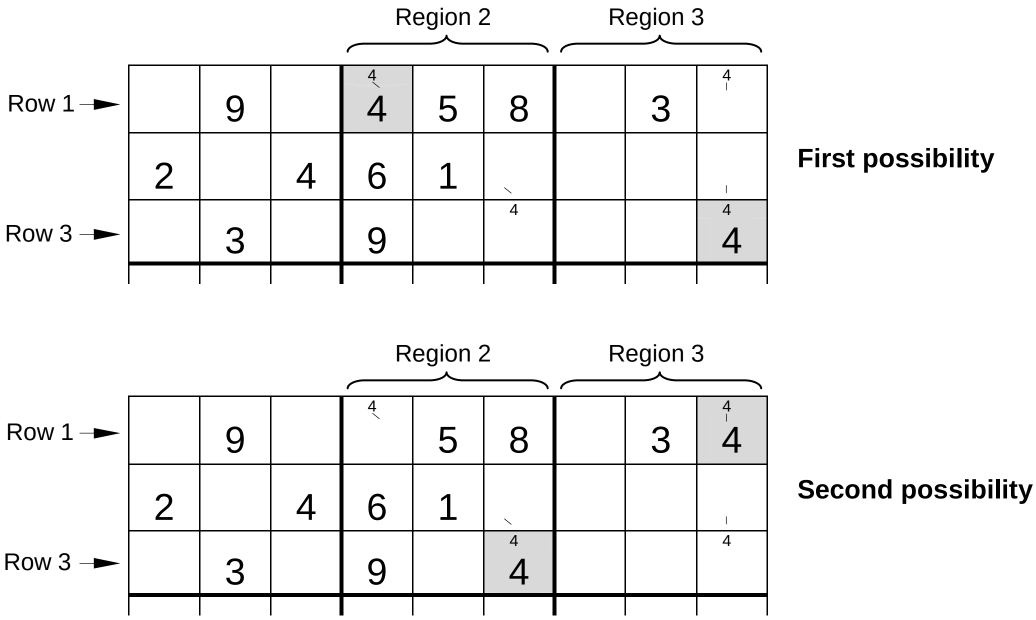

Figure 9.

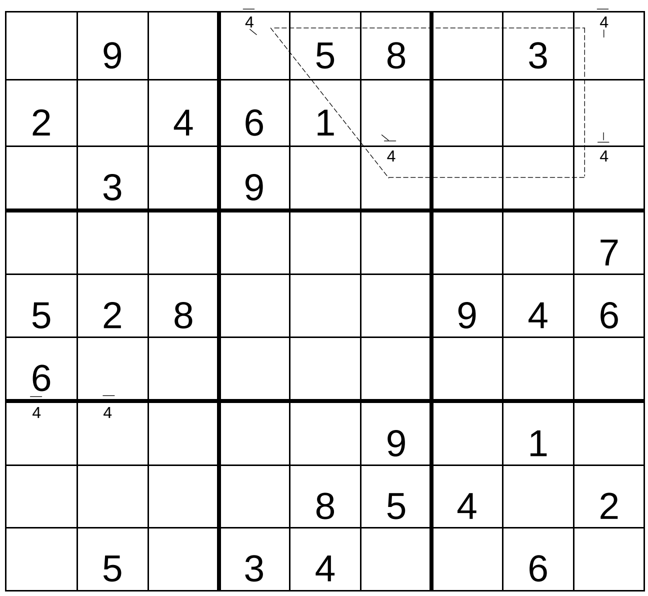

Figure 10.

The x-wing consist of four empty cells linked together in a closed circuit. Always involve 2 regions and 2 rows and/or columns. With an x-wing we have the advantage of having enough information grouped. In this case, besides the effects seen previously with links, the new situation provides the following:

- When placing a number different of 4 in any of the four cells of x-wing, we can automatically locate two new numbers 4.

- When placing a number 4 in any of the four cells of x-wing, we can automatically locate another number 4.

- New links of x-wing stablish new scanning directions.

We must make it clear that the links that make up an X-wing do not differ at all from simple links. Are the same, and everything seen above for simple links is applicable to x-wing links. All that happens is that by being grouped, they allow the information provided is "concentrated" and that, of course, has its advantages.

Another interesting consequence of x-wing of the fact that there are 2 regions and 2 rows and/or columns involved is as follows. For example, in this case involves two regions (2 and 3) and two rows (1 and 3). That means there will be a third region next to the above (region 1) in which the number of x-wing itself will never be in the rows above and therefore in that region can only be in the left row (row 2). In this case, in region 1, row 2 and from the beginning we have placed the number 4, but may not have been there.

Seen essentials about the x-wing, back again with scanning of number 4. There was left one single unscanned region, region 4. In Fig. 10 we see that region 4 can be scanned by two numbers 4. One of them located on the right (region 6) scans three cells already full, which unfortunately doesn’t contribute anything. The other, located above (region 1), scans 2 empty cells, so that definitely we are left with 3 cells without scanning.

As said before, if after region scanning 3 cells are empty without ruling out, directly we can not locate or pencilmark, but sometimes we can get some clues: if empty cells not ruled out are aligned vertically or horizontally, are also setting a scan direction. If not aligned, it is possible to scan and pencilmark a link in rows or columns that cross that region, particularly in wich share whith it a single empty cell not ruled out.

In this case, the 3 cells are not aligned. Row 6 intersects region 4 and shares with it a cell not ruled out (cell in row 6 and column 2). However, in row 6 is not possible to pencilmark another link of number 4 as the scanning of all this row shows that there are too many empty cells.

Column 1 intersects region 4 and shares with it a cell not ruled out (cell in row 4 and column 1). If we make a scanning of number 4 on the whole column 1 we get the pleasant surprise that there are only two cells unscanned, so that we can pencilmark a new link in this column (Fig. 11).

	9		4	5	8		3	4
2		4	6	1				
	3		9		4			4
4								7
5	2	8				9	4	6
6								
4	4				9		1	
				8	5	4		2
	5		3	4			6	

Figure 11.

As we see, besides scanning on regions, which is more intuitive, you can also scan on rows and columns. The fact of finding a region with 3 cells without ruling out has led us to try luck scanning a row and a column, and so we have achieved a new link of number 4.

Number 4 is already completely scanned. We have obtained all the information at this time it could provide. We have introduced many concepts, but from now everything will be easier.

Scanning of number 5

At this point we are supposed beginning to understand what the hell is a link, how to get it and what it does, which henceforth and to speed up, we will pencilmark them without much explanation.

The fundamental idea is that with every number scanned, we will get used to obtain as information as possible and also that this information must to be evident on the board. We must not obsess too much with this either, because the clock is not going to stop. At first is usual that some link escape our notice, which is not a major problem.

We continue with the scanning of number 5. From the table of Fig. 11 we see that for example, scanning horizontally across region 9 we have two free cells that allow to pencilmark a new link (Fig. 12).

	9		(4)	5	8		3	(4)
2		4	6	1				
	3		9		(4)			(4)
(4)								7
5	2	8				9	4	6
6								
(4)	(4)				9	(5)	1	(5)
				8	5	4		2
	5		3	4			6	

Figure 12.

More things. Scanning horizontally and vertically on region 1 we get one free cell, which can only be occupied by a 5. We place it shading on Fig. 13.

	9		(4)	5	8		3	(4)
2		4	6	1				
	3	5	9		(4)			(4)
(4)								7
5	2	8				9	4	6
6								
(4)	(4)				9	(5)	1	(5)
				8	5	4		2
	5		3	4			6	

Figure 13.

Finally, we find a new link from 5 scanning vertically and horizontally on region 5, as shown in Fig. 14.

	9		(4)	5	8		3	(4)
2		4	6	1				
	3	5	9		(4)			(4)
(4)			(5)					7
5	2	8				9	4	6
6			(5)					
(4)	(4)				9	(5)	1	(5)
				8	5	4		2
	5		3	4			6	

Figure 14.

It is no longer possible to get more information about number 5, at least for now.

Scanning of number 6

Barrido del número 6

From Fig. 14, scanning vertically and horizontally on region 1, there is an empty cell left and we can place a definitive 6.

The same occurs in region 8 where another 6 is placed. In Fig. 15 both are shown shading.

	9	6		5	8		3	
2		4	6	1				
	3	5	9					
								7
5	2	8				9	4	6
6								
				6	9		1	
				8	5	4		2
	5		3	4			6	

Figure 15.

We will be able to place more numbers 6 in regions 3, 7 and 5 as shown in Fig. 16.

	9	6		5	8		3	
2		4	6	1				
	3	5	9			6		
					6			7
5	2	8				9	4	6
6								
				6	9		1	
	6			8	5	4		2
	5		3	4			6	

Figure 16.

All nine numbers 6 possible are placed yet. In future we can forget about this digit because it is already full.

Scanning of number 9

From Fig. 16, with digit 9 we obtain more information. A link in region 3, another in region 5 and another one in region 9 (Fig. 17).

If we pay attention to new links marked in Fig. 17 we see that in regions 3 and 9 there is an x-wing and therefore we will also pencilmark the links that close the circuit (Fig. 18).

In addition, scanning vertically number 9 on region 4 we have three free cells that are not aligned. There are therefore able to find something else. Column 1 shares with that region one of those cells. If we scan number 9 on that column there are 3 free cells left, which contributes nothing.

Now we check row 6 that also shares one of the three initial free cells. Scanning number 9 on the entire row 6 and fortunately we found only two free cells, which allow pencilmark a new link as shown in Fig. 18.

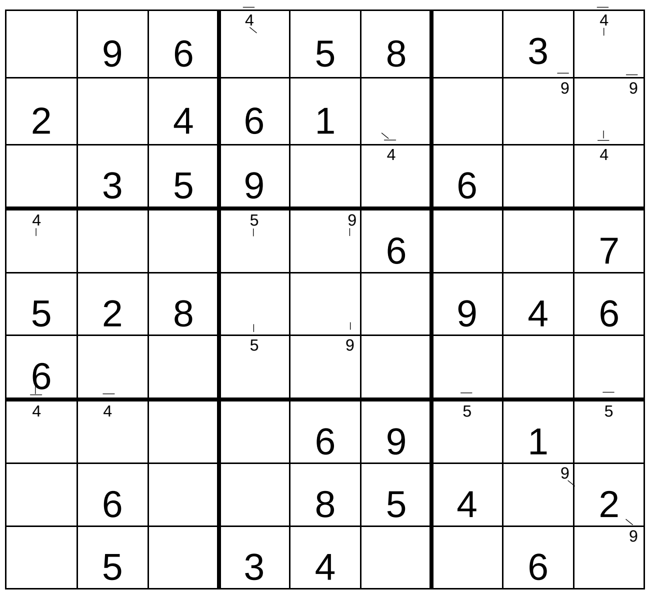

Figure 17.

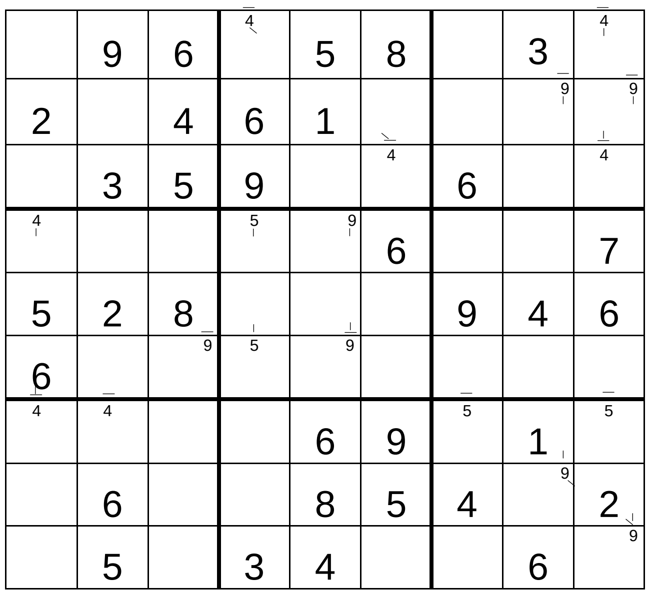

Figure 18.

Now we have used up all the possibilities offered by number 9.

Scanning of number 2

Directly we find links of number 2 in regions 3, 7 and 8 (Fig. 19).

From Fig. 19 we can see another x-wing in regions 7 and 8 which is marked in Fig. 20.

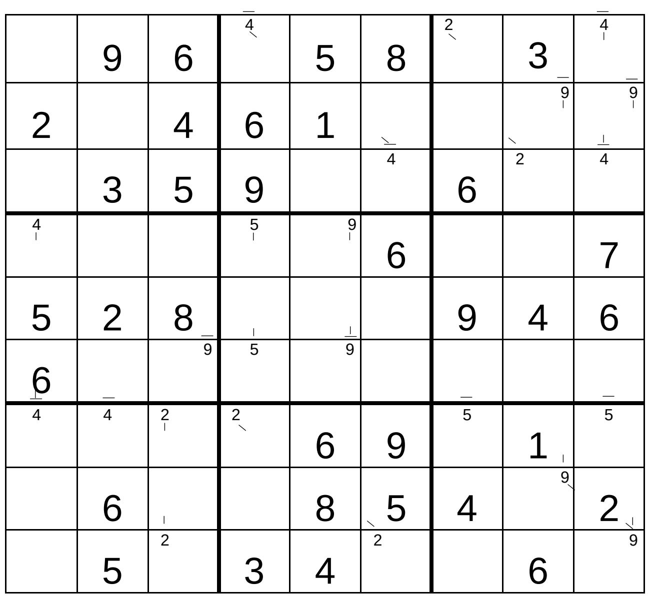

Figure 19.

As the scanning on region 2 provides three free cells that are not aligned, try our luck. Indeed, scanning row 1 we obtain 2 free cells that allow us to add a new link in Fig. 20.

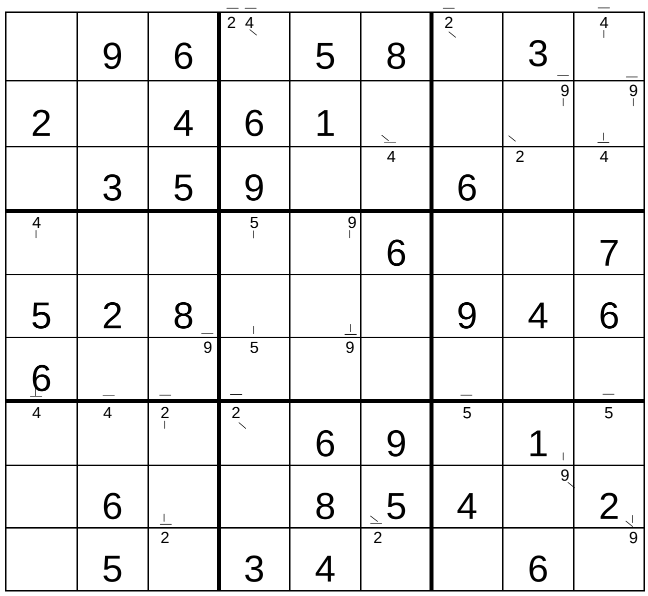

Figure 20.

Scanning of number 3. Naced pairs

From Fig. 20 and scanning over region 2 there is a free cell, and therefore we can place a digit 3, shown shaded on Fig. 21.

In region 9, scanning horizontal and vertically we find a new link (Fig. 21).

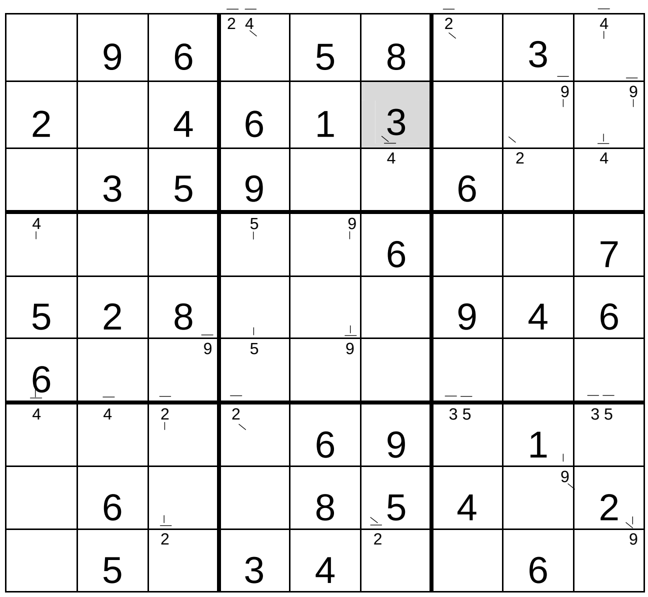

Figure 21.

Before continue scanning number 3 is obvious an important detail in Fig. 21:

In the two empty cells at the upper part of region 9 we have a double link of digits 3 and 5. Since there is a link of 3, the final number 3 definitely could be placed in the left cell or in the right cell. In the same way as there is a link of 5, the final number 5 definitely could be placed in the left cell or in the right cell.

Conclusion: there are only two possibilities, if on the left is 3, on the right is 5, and vice versa, if on the left is 5, on the right is 3. There is no other possibility.

We just found the first **naked pair**. Each one of the two cells that makes up the naked pair has two links, but also has two candidates, in our case 3 and 5.

We had initially declared that naked pairs are represented by underlined numbers and twin strokes, pointing to the direction of the other cell. We can see it in Fig. 22. No need to maintain the lines over the numbers.

Let's review the consequences of naked pairs:

- Naked pairs are links and candidates alike. All properties of the links are also applicable to naked pairs.

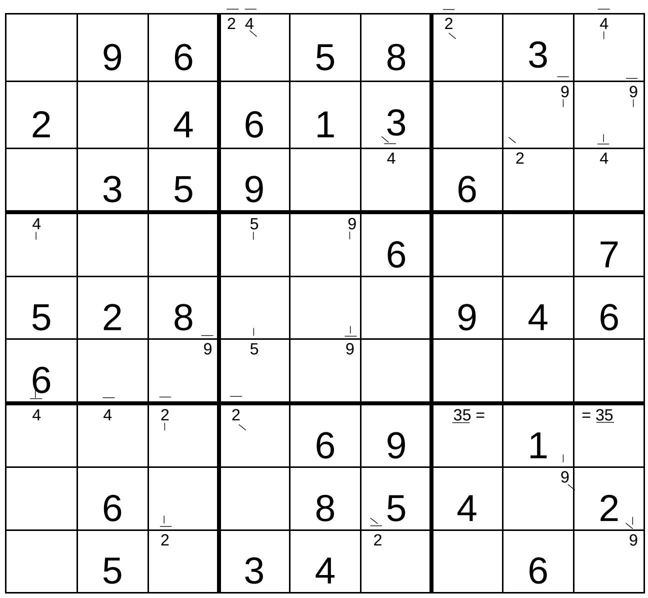

Figure 22.

- Being as well candidates (thus underlined) we know that in each of the cells in a naked pair there are only two possible candidates, in this case 3 and 5.

- The groups to which naked pairs belong can not contain other cells with either of the two candidates of those naked pairs. In our case, in region 9 and also in row 7 is impossible to place a 3 or a 5 on any other empty cell.

- When you locate a definitive number in a cell of a naked pair, we can automatically locate the other candidate in the other cell.

- The naked pairs are not deleted or modified, are waiting to know which of the only two options is correct to place the final numbers in both cells. You could say they are "untouchable”.

- When a group has only two empty cells, these are a naked pair.

- In general, having marked naked pairs on a sudoku provides valuable information, since in many cases are equivalent almost to have numbers already located. Within a group, naked pairs "fill" empty cells.

Examined the novelty of naked pairs, we will continue with the scanning of number 3.

From Fig. 22 we scan vertically region 5. There are only three free cells left, which are also aligned and therefore, as stated earlier, are making up a scanning direction. Apparently we don´t get so much, but if we look closely it is noteworthy that the row 5 has only three empty cells. We do a counting on that row and find that it´s lacking the numbers 1, 3 and 7. By chance, two of the empty cells are scanned away by the number 3. Definitely, number 3 in row 5 can only be in the central cell. We place it in Fig. 23.

	9	6		5	8		3	
2		4	6	1	3			
	3	5	9			6		
					6			7
5	2	8		3		9	4	6
6								
				6	9		1	
	6			8	5	4		2
	5		3	4			6	

Figure 23.

Now, row 5 has only two empty cells, which can only contain the numbers 1 and 7. At the moment we don´t know in what order are they placed, but it is clear that they stablish a naked pair as mentioned above (group with two empty cells). So that is represented in Fig. 24.

Let’s continue. In region 6 we have three free cells not aligned after scanning. Try our luck and see that scanning the number 3 on the column 9 we have two free cells. The new link is shown in Fig. 25.

Now in Fig. 24 we are going to scan the number 3 on region 7, vertically with the number 3 at region 1 and horizontally with the number 3 at region 8 and the link of 3 (which also forms a naked pair) at region 9. We can do this because links (and therefore also naked pairs) provide scanning directions as stated above. Thus in region 7 there are three free cells that allow to pencilmark a new link (Fig. 25).

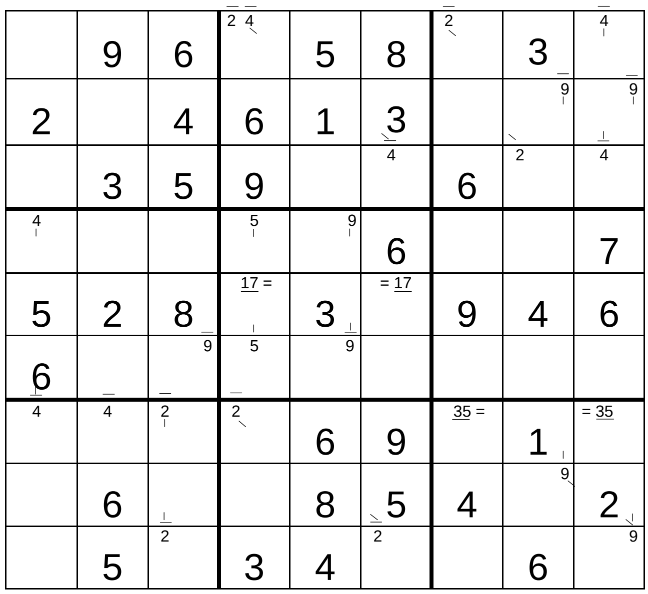

Figure 24.

Finally, we have to scan region 4. In doing so (see Fig. 24) remain free three cells that are not aligned. Fortunately, column 1 is scanned in all cells (including one by the naked pair in region 9) except two. So, we will pencilmark the appropriate link as shown in Fig. 25.

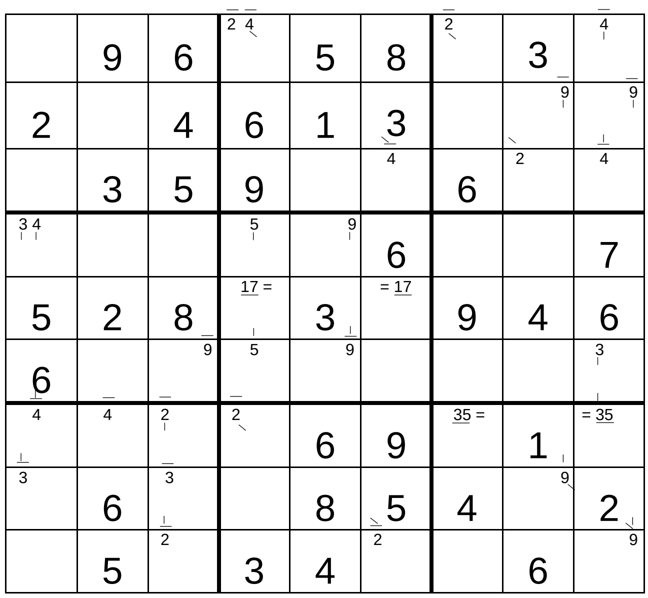

Figure 25.

Scanning of number 8

At this point you can say you know everything important about links method. As of now we have to implement the methodology and little else. We continue scanning the remaining numbers. Let's consider eight.

From Fig. 25, the horizontal scanning of digit 8 on region 1 makes up two free cells. We pencilmark the corresponding link (Fig. 26).

Region 9 is also scanned horizontally, remaining four free cells, but beware, two of them are naked pairs and so are "untouchables", almost definitive numbers. In the naked pair on region 9 only to decide where are placed numbers 3 and 5 is left. Thus, for the scanning of No. 8 on region 9 are only 2 free cells, in which we pencilmark the link as shown in Fig. 26.

By scanning on region 5 occurs something similar. Three cells are free, but one of them belongs to a naked pair and is not considered. Actually we have two free cells, so we pencilmark the corresponding link (Fig. 26).

And the scanning on region 7, which is involved the link just marked on region 9, provide us two free cells and therefore a new link as shown in Fig. 26.

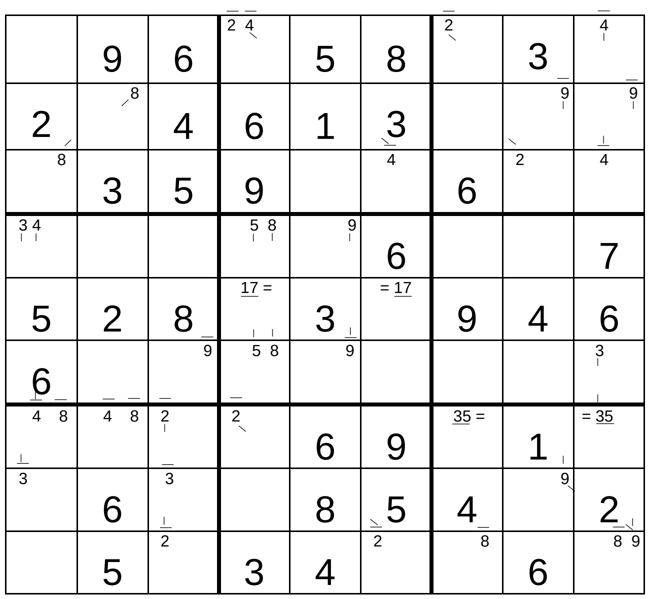

Figure 26.

The astute reader will already have noticed that at the table of Fig. 26 has been formed some new naked pair. Specifically there are two, one in region 5 and the other in region 7. Both are marked in Fig. 27.

As said before, to solve a sudoku is simply a sequence of actions leading to consequences, which lead to more consequences, and so on, until the puzzle is solved entirely.

As said also before, within a group, naked pairs "fill" empty cells. Look at the row 7 (Fig. 27). It has three definitive numbers (6, 9 and 1) and four cells with naked pairs. What remains? We have two empty cells. Which numbers can be placed at these two cells? 2 and 7. We have another naked pair (Fig. 28).

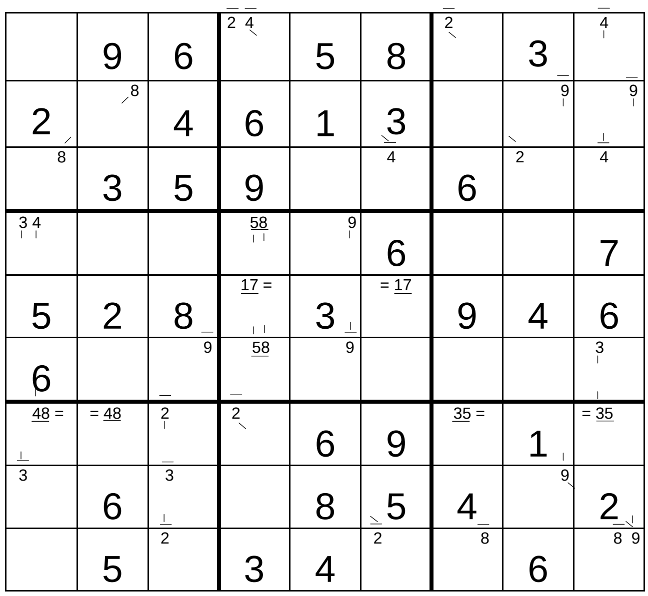

Figure 27.

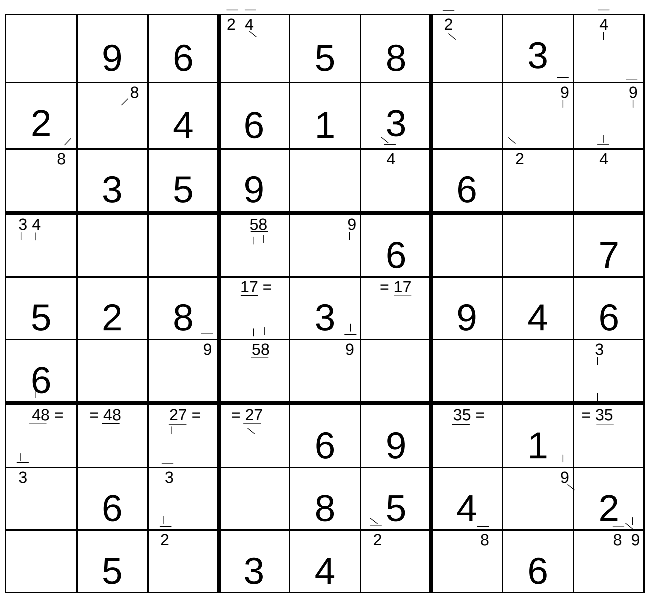

Figure 28.

Scanning of number 1

Scanning number 1, from Fig. 28 we obtain directly a link in region 1 and another link in region 8 (Fig. 29).

Now we can scan vertically on region 7 for a new link, which also set up an x-wing with the link of region 8. Moreover, a naked pair is made up in row 9 with numbers 1 and 2.

As a consequence, region 4 can be scanned with the links found in regions 1 and 7 for obtain other new link.

Furthermore, if we scan region 6, three free cells are left that allow us to pencilmark another link in row 4. Similarly, in region 3 are also three unscanned cells, so try our luck we can find another link in row 3.

All that is shown in Fig. 30.

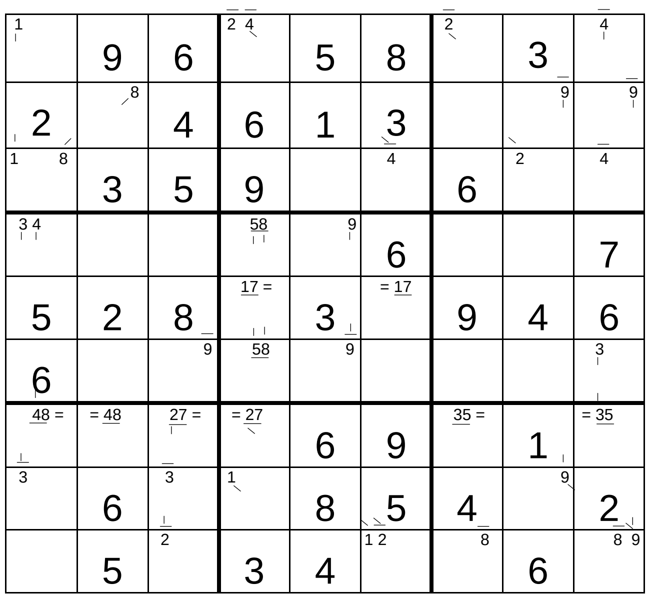

Figure 29.

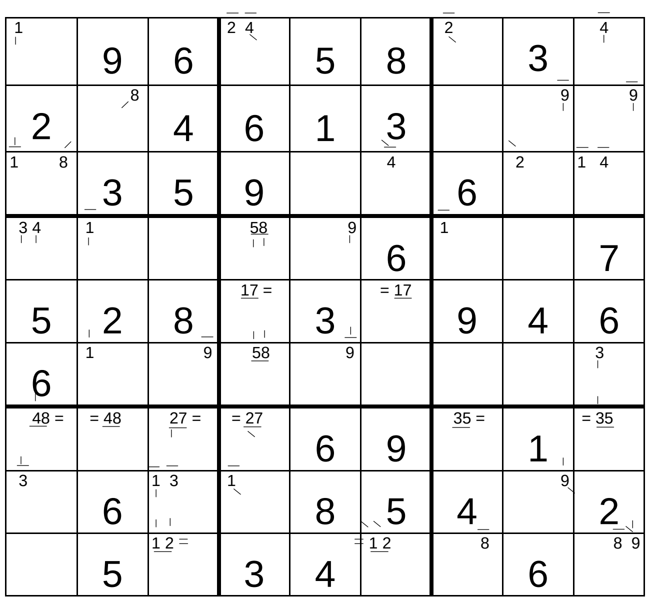

Figure 30.

Scanning of number 7

The last digit that remains sweeping is 7. Recalling that naked pairs "fill" groups, we get two links in regions 4 and 9 (Fig. 31).

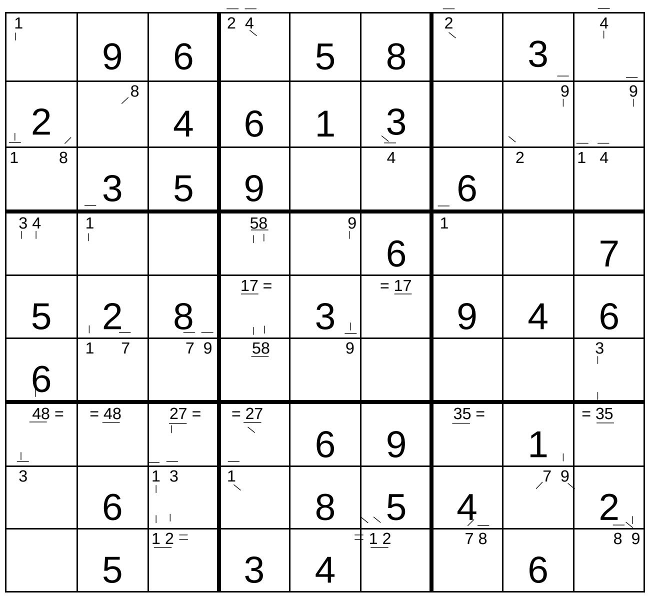

Figure 31.

More scannings and countings

So far we have scanned one by one the nine numbers, thereby obtaining a significant amount of information. On the board, we have new definitive numbers and many pencilmarks such as simple links, x-wings, and naked pairs. All this means that anytime a new track may cause the result that any pencilmark is solved allowing to place more definitive numbres and triggering more impact in other pencilmarks, which in turn leads to other consequences, thereby facilitating the speedy resolution of sudoku, at least in most cases.

What we need is to find the "ignition spark", the trigger that starts the chain reaction. And that's what we do now. How? By countings and more scannings, and also being alert and using, as always, common sense.

We begin in Fig. 31 with the easiest, with groups with fewer empty cells. For example, the column 5 has 3 of them, which should contain the missing numbers after counting, which are 2, 7 and 9. However, two of these cells may not have the number 7 by being scanned horizontally by it and one of its links, or because in region 5 there is a naked pair involving number 7 yet.

Consequently, we place a definitive 7 and pencilmark a naked pair in the two remaining cells (Fig. 32).

Now in region 5 are two definitive numbers and three naked pairs, leaving an empty cell. Number 4 can be located there (Fig. 33).

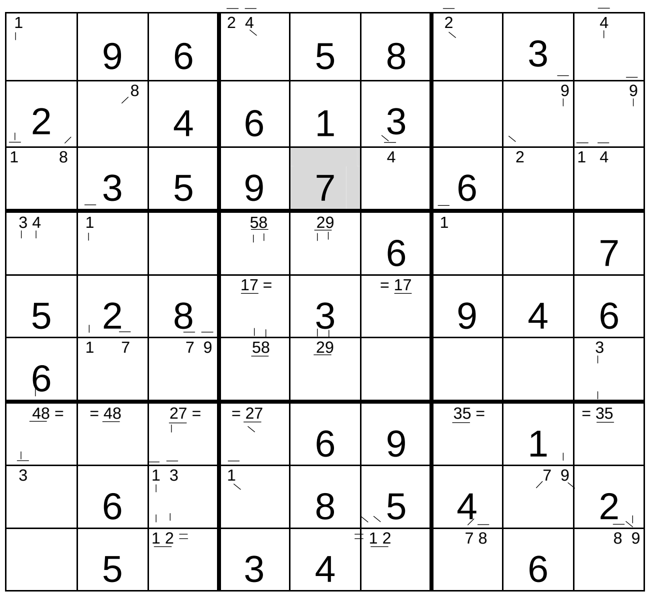

Figure 32.

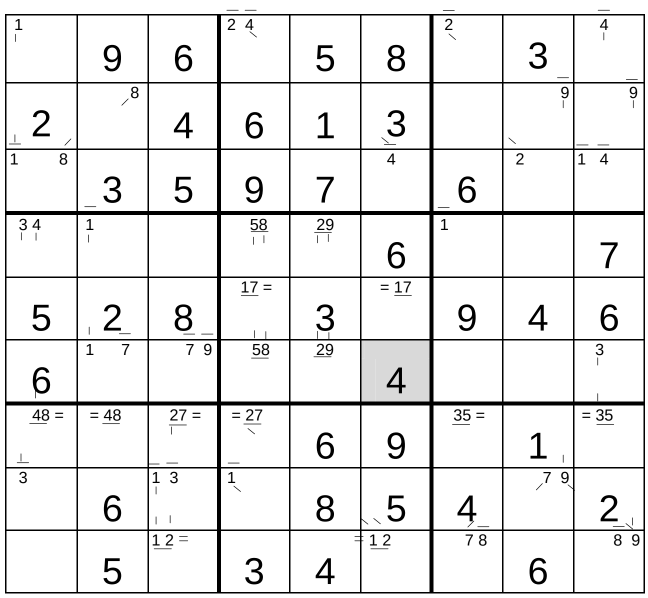

Figure 33.

This new number 4 opens new possibilities. In the same column 6 there is one of the four elements of the x-wing of number 4 in regions 2 and 3. We can solve the x-wing and locate two new fours.

This also causes the resolution of other two links, the number 2 (row 1) and the number 1 (row 3), which gives rise to locate more definitive numbers and also to solve a new link, the number 8 in region 1, with another definitive number. This is illustrated in Fig. 34.

The last 8 affects naked pair of region 7, which is also determined, affecting furthermore the links of 4 and 3 (column1) that are solved.

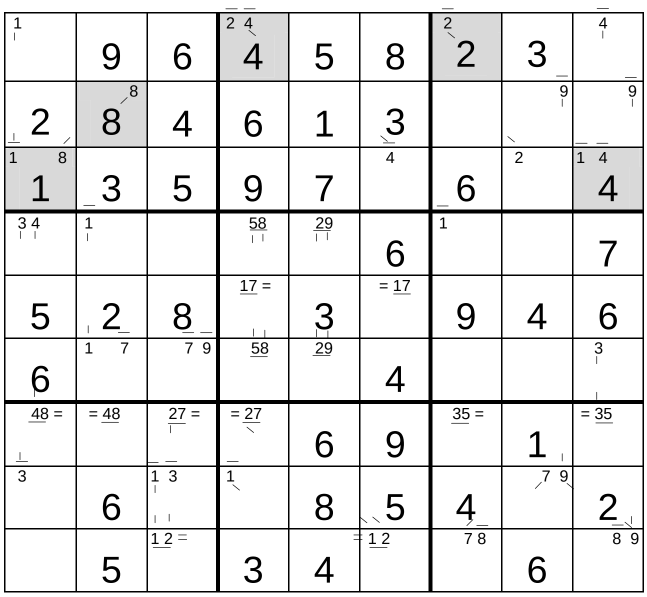

Figure 34.

On the other side region 1 has a single cell empty left, that after counting must be occupied by number 7. This done, row 1 is also with an empty cell, which allows us to locate digit 1. The same goes to region 2, which lacks just number 2. Once positioned, third row is also an empty cell, which will complete with number 8 (Fig. 35).

Solved or residual pencilmarks are no longer valid and should be deleted (Fig. 35).

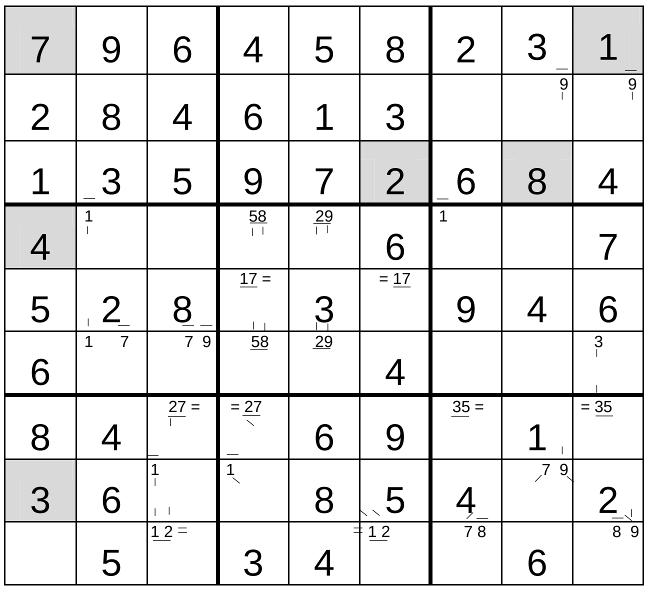

Figure 35.

More. In Fig. 35, column 1 is left with an empty cell. Counting, we can place number 9.

We have to verify whether the new placed number affects the groups to which it belongs and, indeed, in the same row we see a link of 9 which is actually part of an x-wing in the regions 9 and 3. Furthermore, in region 9 we can also solve two more links, enabling to locate definitives 7 and 8 (Fig. 36).

7	9	6	4	5	8	2	3	1
2	8	4	6	1	3			9
1	3	5	9	7	2	6	8	4
4	(1)		(58)	(29)	6	(1)		7
5	2	8	(17)	3	(17)	9	4	6
6	(1 7)	(7 9)	(58)	(29)	4			(3)
8	4	(27)	(27)	6	9	(35)	1	(35)
3	6	(1)	(1)	8	5	4	9	2
9	5	(1 2)	3	4	(1 2)	7	6	8

Figure 36.

In row 9, Fig. 36, now there are two cells with the naked pair of numbers 1 and 2. Fortunately, number 2 of region 2 affects one of these cells and allows us to locate and complete row 9.

Also, the previous naked pair had other links, allowing to fill regions 7 and 8 and solve another naked pair in region 5 and two links of the numbers 7 and 1 in region 4 (Fig. 37).

7	9	6	4	5	8	2	3	1
2	8	4	6	1	3			9
1	3	5	9	7	2	6	8	4
4	1		(58)	(29)	6			7
5	2	8	1	3	7	9	4	6
6	7	(9)	(58)	(29)	4			(3)
8	4	7	2	6	9	(35)	1	(35)
3	6	1	7	8	5	4	9	2
9	5	2	3	4	1	7	6	8

Figure 37.

The end is near. Region 3 has two empty cells. Counting, note that numbers 5 and 7 have to be located. However, 7 of region 9 scans one of the two cells, so we locate and complete region. This affects the naked pair in region 9, which also is solved, and column 9 which become full. As in region 4 to locate 3 and 9 was needed, we can now complete it (Fig. 38).

7	9	6	4	5	8	2	3	1
2	8	4	6	1	3	5	7	9
1	3	5	9	7	2	6	8	4
4	1	3	58	29	6			7
5	2	8	1	3	7	9	4	6
6	7	9	58	29	4			3
8	4	7	2	6	9	3	1	5
3	6	1	7	8	5	4	9	2
9	5	2	3	4	1	7	6	8

Figure 38.

To round off, in Fig. 38 we can solve the naked pair of the numbers 2 and 9 through the last 9 which had located. Then we scan region 6 with number 2 and locate 2 and 5 to complete column 8.

Finally, we can complete columns 7 and 4. The final result is shown in Fig. 39.

7	9	6	4	5	8	2	3	1
2	8	4	6	1	3	5	7	9
1	3	5	9	7	2	6	8	4
4	1	3	5	9	6	8	2	7
5	2	8	1	3	7	9	4	6
6	7	9	8	2	4	1	5	3
8	4	7	2	6	9	3	1	5
3	6	1	7	8	5	4	9	2
9	5	2	3	4	1	7	6	8

Figure 39.

8.- SUDOKU #2

Here is another example (Fig. 40).

6	3				2			
	9	7					1	
3	7	4				9		
		8		9		2	6	
				8			7	
9		1			8			4
4	8				9	7		
		6		5				

Figure 40.

It follows that 1 appears 2 times; 2, 2 times; 3, 2 times; 4, 3 times; 5, 1 time; 6, 3 times; 7, 4 times; 8, 4 times; 9, 5 times. Therefore, a proper order to start the scans will be: 9, 7, 8, 4, 6, 1, 2, 3, 5.

Scanning of numbers 9, 7 and 8

Next step is scanning the nine numbers in the order given above. Let's start with 9 (Fig. 41).

Likewise we scan number 7 (Fig. 42):

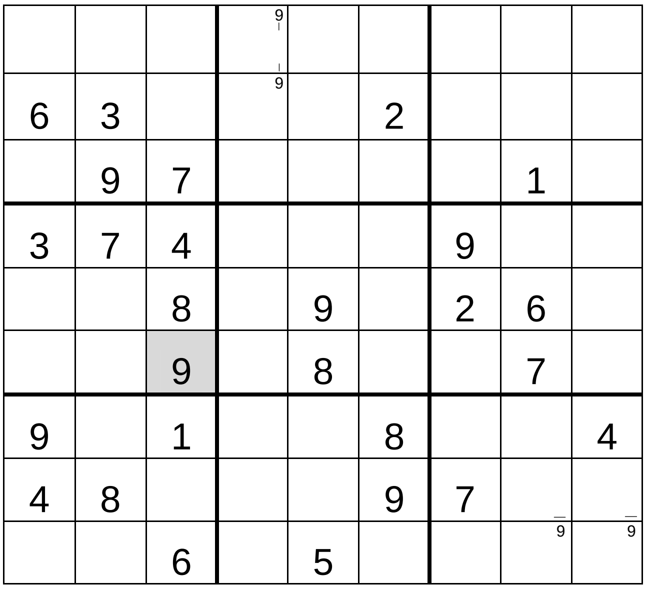

Figure 41.

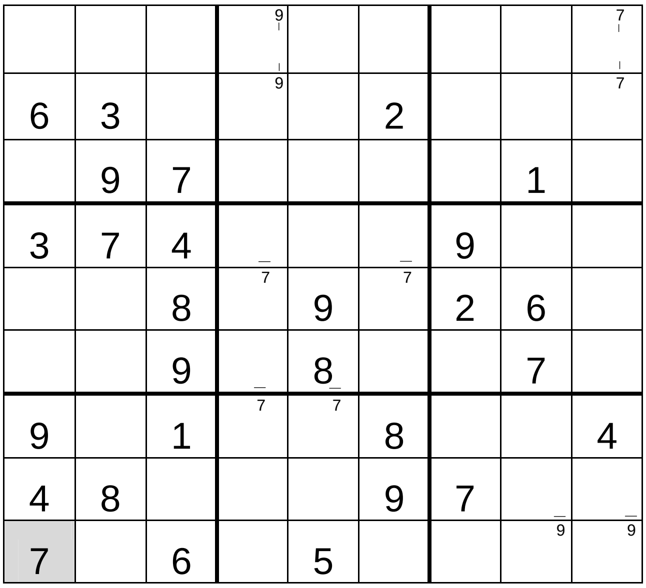

Figure 42.

Now let's sweep the 8, which provide us two links (Fig. 43).

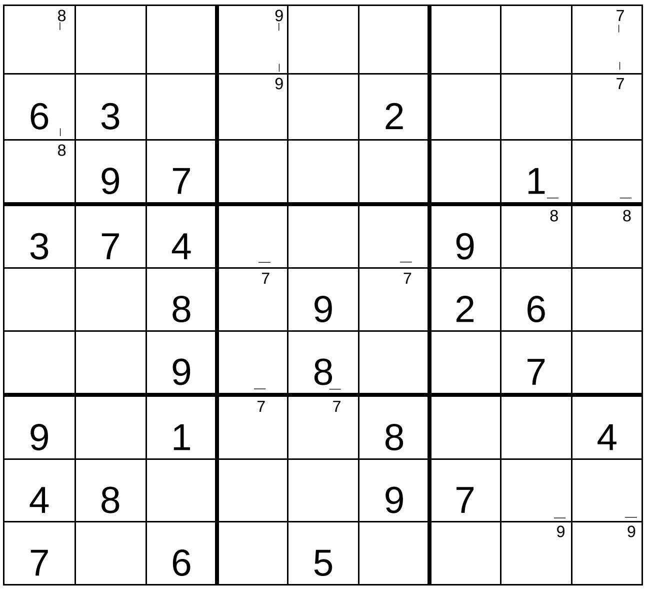

Figure 43.

Scanning of number 4

Scanning of number 4 allows us to locate three definitive numbers and pencilmark two links (Fig. 44).

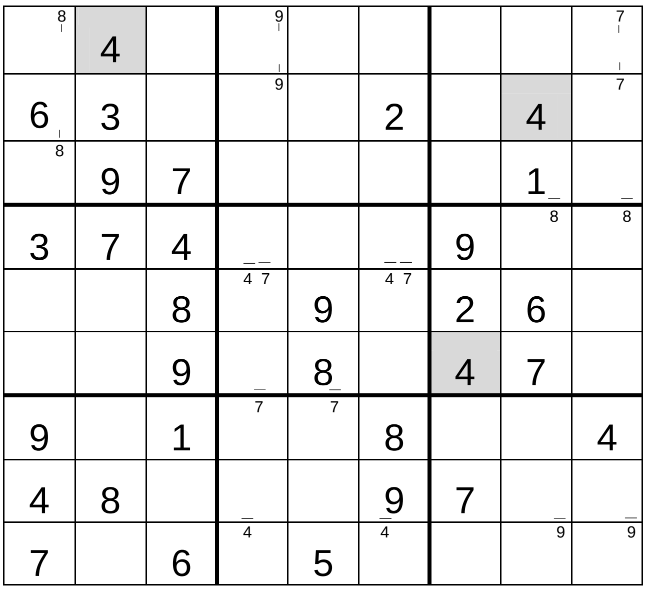

Figure 44.

In addition, we have found a naked pair (region 5) and an x-wing, as shown in Fig. 45.

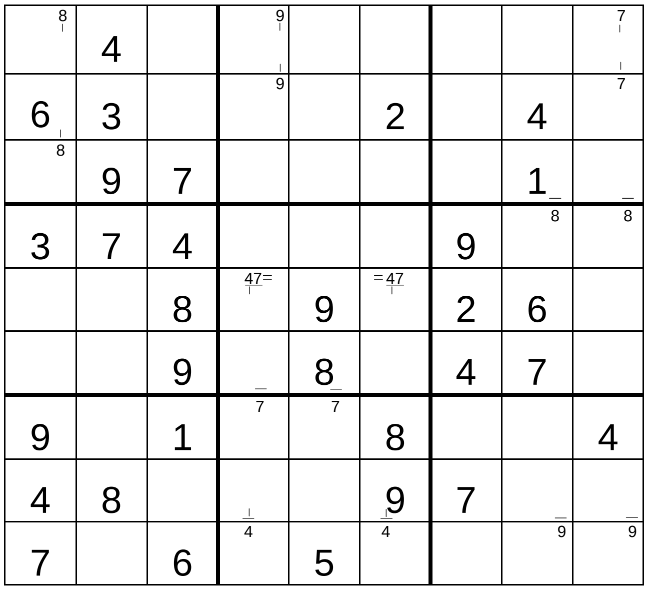

Figure 45.

New vertical links from 4 to close x-wing circuit will bring other consequences. Horizontal and vertical scanning of number 4 on region 2 allow to get another definitive 4 (shaded cell). Resultant board is shown at Fig. 46.

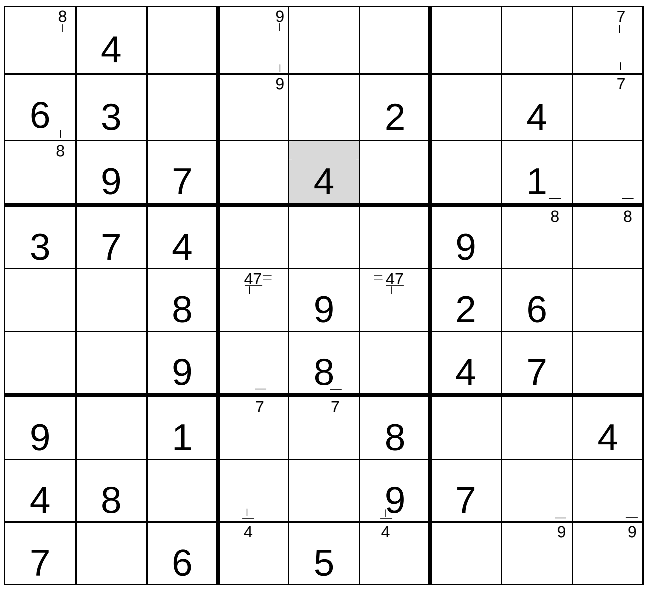

Figure 46.

Scanning of numbers 6 and 1

We will focus now on 6 and 1. Let's check how they can contribute to bring new data.

Scanning number 6 we can place a definitive on region 4 and pencilmark a link on region 9.

Scanning number 1 on region 1 we place a definitive in the cell where there was a link of number 8. This also allows us to place an 8 in this region. Besides, the new 1 scans region 4 allowing to place another definitive 1 (Fig. 47).

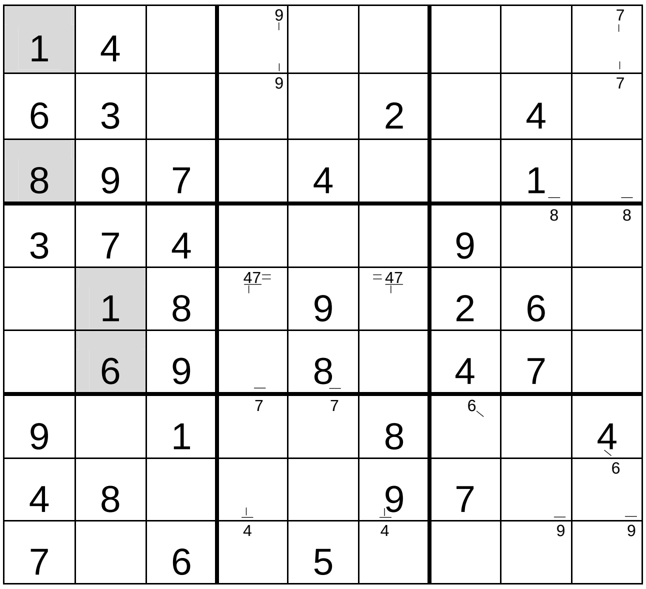

Figure 47.

But let's make a stop along the way. In Fig. 47 it is noteworthy that region 1 now has only two empty cells. For counting, these cells may contain just 2 or 5. However, number 2 at region 2 requires to place 2 above and 5 below. We write them and then we have two new definitive numbers, whose shaded cells can be seen in Fig. 48.

In addition, another look at the board shows that third column has now only an empty cell, which cannot be other than number 3. We place it also in Fig. 48.

But there's more. Region 7 has two empty cells left. Counting, we realize that can only be 2 or 5. At the same time we find that there is a 5 in region 8 that scans one of these two cells, the lower one. So we can place both definitive numbers, shaded in Fig. 49.

Likewise, region 4 is completed by placing 2 and 5 (Fig. 49).

Finally, at row 5, despite the fact that after locating previous number 5 there are three remaining empty cells, two of them form a naked pair, implying that the remaining cell can only be a 3. It is also located in Fig. 49.

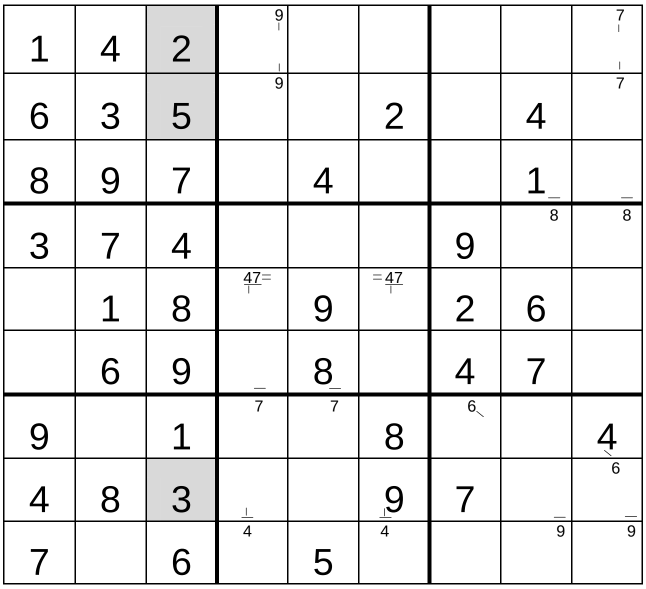

Figure 48.

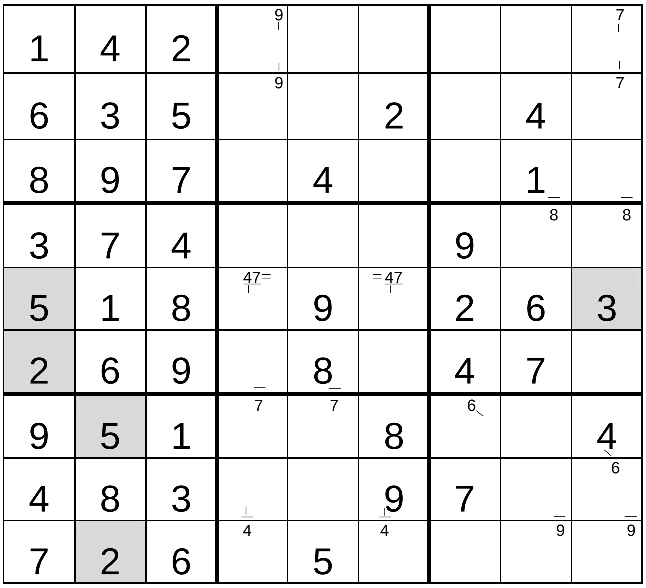

Figure 49.

Scanning of numbers 2, 3 and 5

We still have to scan numbers 2, 3 and 5. Scanning 2 we find a definitive digit on region 3 and two new links on regions 9 and 5 (Fig. 50).

By doing scans of 3 it´s found a link on region 5, and finally, scanning number 5 we pencilmark a new link on region 9 (Fig. 50).

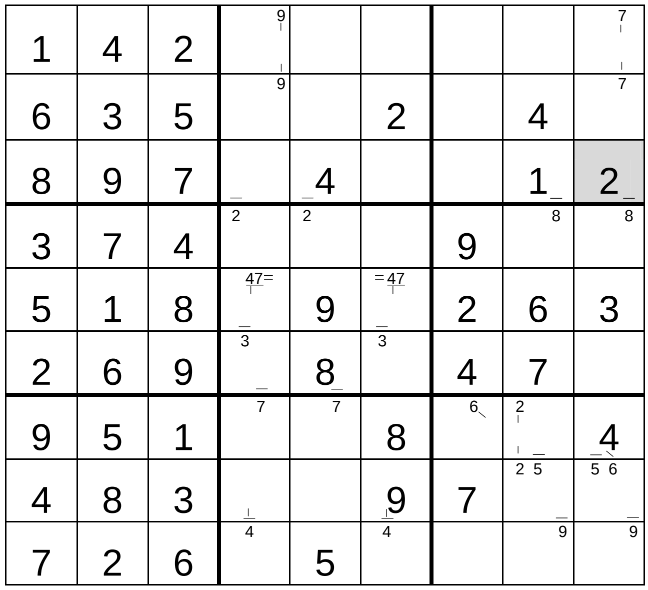

Figure 50.

More scannings and countings

Looking at Fig. 50 we´re going to look for groups that contain a few empty cells, as they are the easiest to complete. With an empty cell no results are found. With two, is row 5, which already has a naked pair and at the moment little can be done. However it´s possible to recount groups containing three empty cells.

On region 6 there are three empty cells, which must be occupied by numbers 1, 5 and 8. As 1 of region 3 scans one of these cells, we can link it on the other two. This new link allows scanning region 9 to find a new definitive 1, which shaded cell is shown at Fig. 51.

Now we can scan more times number 1 to find more links yet. The link of region 8 is obtained by scanning definitive numbers 1 of regions 7 and 9. The link on region 2 is obtained by scanning numbers 1 of regions 1 and 3. So then it´s established an x-wing (Fig. 51).

Finally, scanning the sixth column and taking into account that naked pair of numbers 4 and 7 is "untouchable", we get a new link of 1 (Fig. 51).

We will make new scans with number 8. In region 9 we get a new link that matches with another one of number 9. The same applies on region 2. Both cases become naked pairs, but the second also had previously another link of number 1 in the lower cell. In that box may no longer be the definitive 1, so that we can place it in the other linked cells and solve the x-wing. The result of all this is shown in Fig. 52.

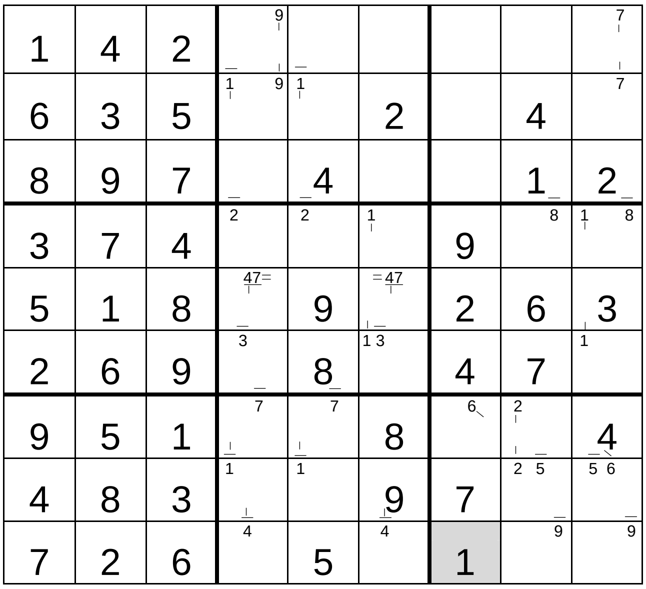

Figure 51.

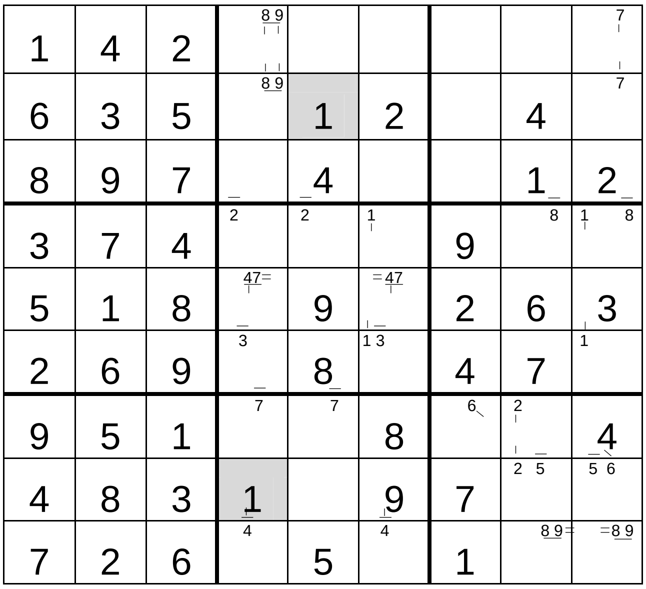

Figure 52.

Counting in a group that previously there are naked pairs, we can consider them as "almost-definitive" or able to "fill" cells, to focus on the remaining free cells. This is what happens in row 9, where the only two free cells may take the values 3 or 4 (Fig. 52). As there is a link of digit 4, by adding the link of 3 we get a new naked pair (Fig. 53).

In the second row of Fig. 52 only numbers 7, 8 and 9 are remaining in the three free cells. But at those free cells, which is at the center (column 7) can not contain the numbers 7 or 9 wich are present in the corresponding column. It contains digit 8, shown shaded in Fig. 53. Furthermore, this new number affects naked pair on region 2, therefore allowing you to place three new definitive numbers completing second row by counting (Fig. 53). For clarity we erase residual marks.

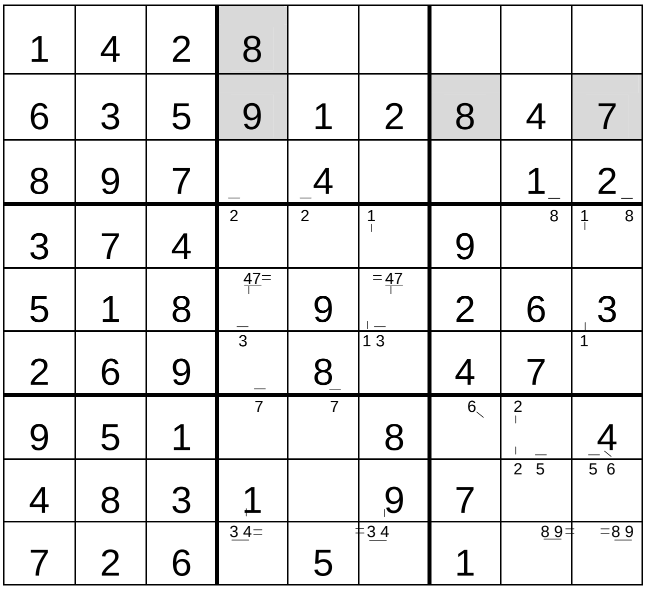

Figure 53.

Lets obtain more links that could help us. In columns 4 and 6 number 3 is linked in two diferent regions, 5 and 8, in the last one by means of naked pair with 4 (Fig. 53). This forms an x-wing of number 3 that we represent with new vertical strokes in Fig. 54.

We can now vertically scan new vertical links of number 3 to region 2 in order to place a definitive 3 (Fig. 54).

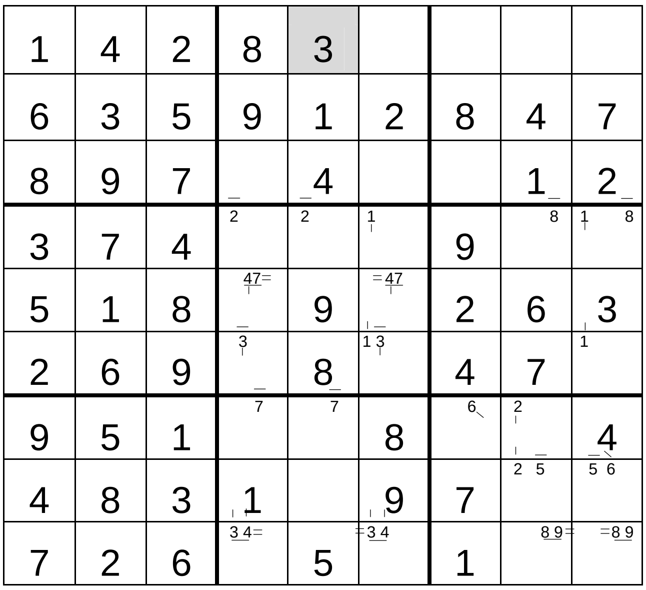

Figure 54.

At column 5 there are 3 free cells (Fig. 54), which can only be occupied by digits 2, 6 and 7. If we scan number 7 on this column remains a unique gap at region 8. As 7 can only be there, we write it and add shading to highlight in Fig. 55. Further we delete link 7, which no longer help us.

In addition, the two empty cells remaining in column 5 provides the naked pair of numbers 2 and 6. As well there are 2 empty cells in the region 8 which means again another naked pair. Both are shown in Fig. 55.

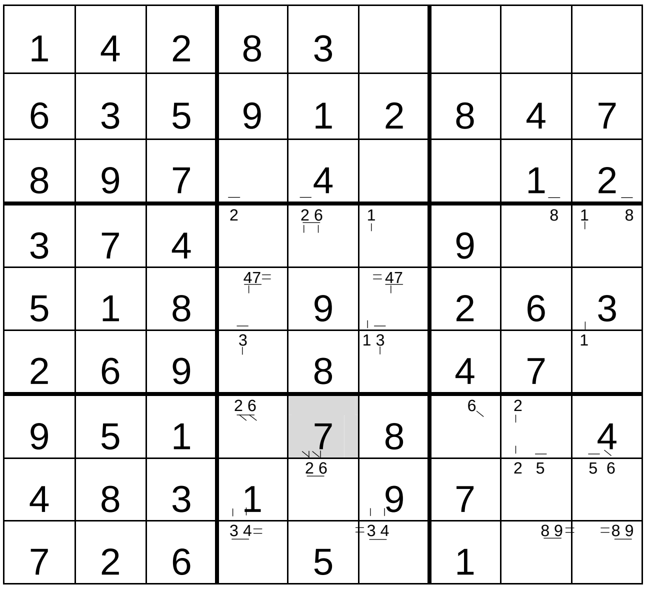

Figure 55.

In addition, scaning number 7 on the region 2 (Fig. 55) we can place another 7. Automatically, the naked pair 4-7 in region 5 is solved (Fig. 56).

As numbers 4 are linked in x-wing with the naked pair 3-4 at region 8, it can be solved (Fig. 56).

Is shown in Fig. 56 that region 2 has only two empty cells that should contain 5 or 6. Since we don´t know where everyone goes, at the moment we will mark the corresponding naked pair (Fig. 57).

This allows us to complete row 3 with the only possible number in the empty cell. For counting, it´s digit 3. (Fig. 57).

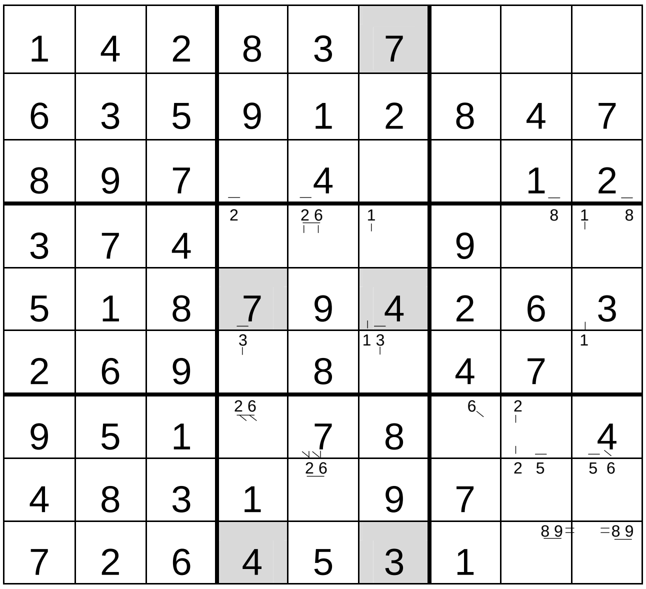

Figure 56.

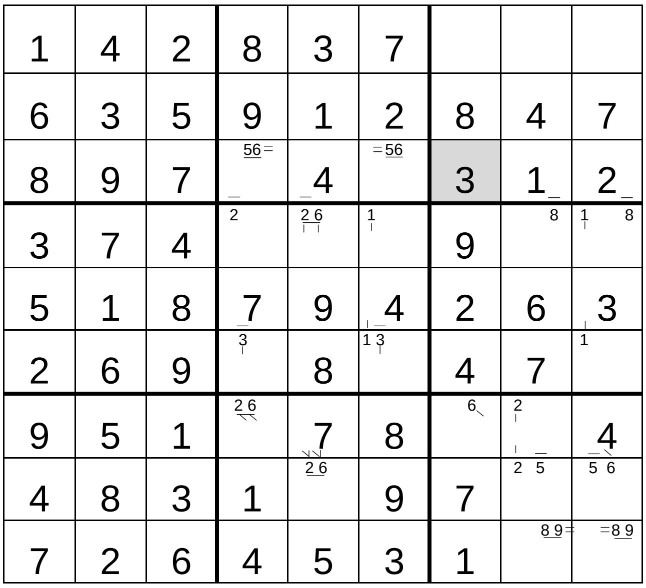

Figure 57.

A general overview at Fig. 57 reveals that column 7 has 2 empty cells. They can contain numbers 5 and 6. But the empty cell of region 9 is scanned horizontally by a number 5, which implies that there goes the 6 and in the other empty cell, digit 5 (Fig. 58). We can also delete the link that affects 6 just written, because it´s no longer useful.

The definitive 6 just written in column 7 allow to solve two naked pairs, starting with located at region 8 which is thus completed and affecting in addition to the one placed at column 5 (Fig. 58).

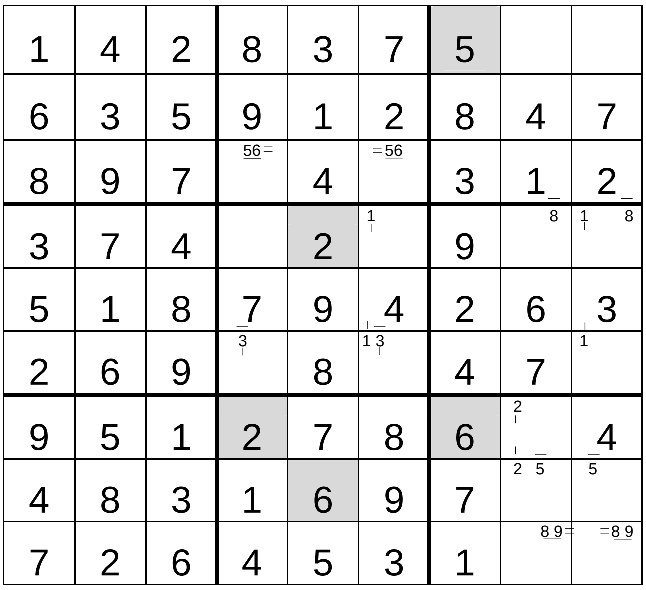

Figure 58.

At region 3 there are two free cells (Fig. 58) that by counting must contain the numbers 6 and 9. One of them is scanned vertically by the 6, which allows to locate both. Moreover, digit 9 just written provides final resolution of the naked pair in region 9 (Fig. 59).

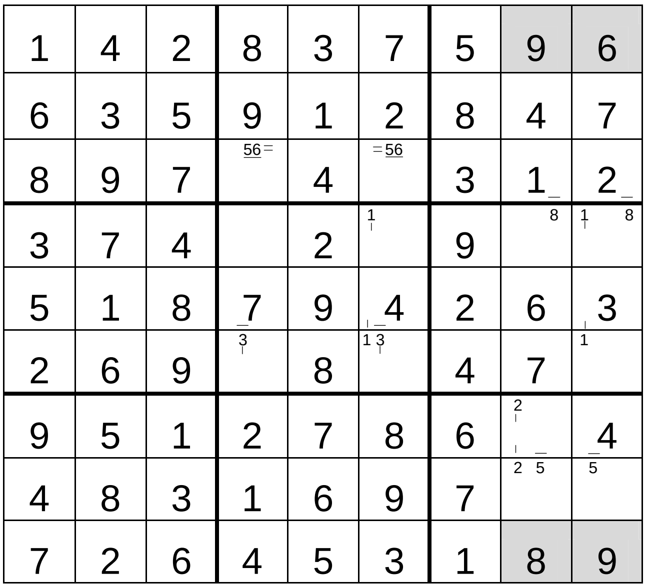

Figure 59.

The link of number 2 in region 9 is solved by scanning horizontally, and consequently also the link of 5, thus completing the entire region (Fig. 60).

1	4	2	8	3	7	5	9	6
6	3	5	9	1	2	8	4	7
8	9	7		4		3	1	2
3	7	4		2		9		
5	1	8	7	9	4	2	6	3
2	6	9		8		4	7	
9	5	1	2	7	8	6	3	4
4	8	3	1	6	9	7	2	5
7	2	6	4	5	3	1	8	9

Figure 60.

From Fig. 60 we can complete column 8 and consequently region 6 (Fig. 61).

1	4	2	8	3	7	5	9	6
6	3	5	9	1	2	8	4	7
8	9	7		4		3	1	2
3	7	4		2		9	5	8
5	1	8	7	9	4	2	6	3
2	6	9		8		4	7	1
9	5	1	2	7	8	6	3	4
4	8	3	1	6	9	7	2	5
7	2	6	4	5	3	1	8	9

Figure 61.

Solving the remaining empty cells is immediate and complete sudoku shown in Fig. 62.

1	4	2	8	3	7	5	9	6
6	3	5	9	1	2	8	4	7
8	9	7	5	4	6	3	1	2
3	7	4	6	2	1	9	5	8
5	1	8	7	9	4	2	6	3
2	6	9	3	8	5	4	7	1
9	5	1	2	7	8	6	3	4
4	8	3	1	6	9	7	2	5
7	2	6	4	5	3	1	8	9

Figure 62.

Author's notes

- This manual was originally written in Spanish. I apologize for the errors of translation.
- Any suggestion or indication towards improving the content, wording or translation errors is welcome.

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